Two-dimensional electron liquid with disorder in a weak magnetic field

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Abstract

We present the effective theory for low energy dynamics of a two-dimensional interacting electrons in the presence of a weak short-range disorder and a weak perpendicular magnetic field, the filling factor $\nu \gg 1$. We investigate the exchange enhancement of the g-factor, the effective mass and the decay rate of the simplest spin wave excitations at $\nu = 2N + 1$. We obtain the enhancement of the field-induced gap in the tunneling density of states and dependence of the tunneling conductivity on the applied bias.

1 Introduction

A two-dimensional electron gas in the perpendicular magnetic field has attracted much attention both from the theoretical and from the experimental points of view. The effects in a strong magnetic field when only the lowest Landau level is occupied have been investigated since the discovery of the quantum Hall effect [1]. Several efforts [2] are made in order to involve the larger filling factors $\nu > 1$ into the problem discussed. However, an existence of the small parameter, namely, a ratio of the Coulomb energy at the magnetic field length to the cyclotron energy has been assumed. In fact, in a weak magnetic field the Coulomb energy at the magnetic field length exceeds the cyclotron energy and some attempts [3] have been undertaken to investigate the case of the large filling factor $\nu > 1$.

The experimental investigation of the tunneling density of states for the system considered was performed at small ($\nu < 1$) [5] and at large ($\nu > 1$) [4] filling factors. In the case of a weak magnetic field ($\nu \gg 1$) the gap in the tunneling density of states has been obtained in the framework of hydrodynamical approach [6]. The progress was made by Aleiner and Glazman [7] who have developed the effective theory for low-energy excitations on the partially filled Landau level at a large filling factors $\nu \gg 1$.

Recently, after the prediction that the unidirectional charge-density wave state occurs at half-filled high Landau levels within the framework of Hartree-Fock theory [8] and the experimental discovery of compressible states with the anisotropic magnetotransport properties in the high mobility systems near the half-filling of the high Landau levels [9], the two-dimensional electron liquid in a weak magnetic field has been studying intensively [10].

In the paper the low energy effective theory for electrons at the partially filled Landau level with the large filling factor in the presence of disorder is developed (Sec.2). As an example, the effect of disorder on the exchange enhancement of the effective g-factor and the simplest spin-wave excitations are discussed in Sec.4. An electron tunneling into the electron liquid is considered in Sec.5. The conclusion is given in Sec.6.

2 Derivation of the effective action

2.1 Introduction

We consider the system of two-dimensional electrons with the Coulomb interaction in the presence of disorder and a perpendicular magnetic field H. The system possesses a partially filled high Landau level with the

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level index $N \gg 1$ equal to the integer part of a half of the filling factor ν, namely, $N = [\nu/2]$. The presence of a random potential which is considered to be short-range results in a broadening of the Landau levels. We imply that the elastic collision time satisfies the condition $\tau_0 \gg \omega_c^{-1}$ where $\omega_c = eH/m$ is the cyclotron frequency with the charge of electrons e and the electron mass m. In this case the broadening of the Landau levels which is of order of $\sqrt{\omega_c \tau_0}/\tau_0$ is much less then the distance between them.

The conventional parameter characterizing the coupling strength of the Coulomb interaction is $r_s = \sqrt{2}e^2/v_F$ with v_F being the Fermi velocity. We assume that electrons are weakly interacting, i.e., $r_s < 1$. In this case we can treat the problem discussed in the random phase approximation. We also assume that the number N is sufficiently large, so the condition $Nr_s \gg 1$ satisfies. This means that the cyclotron radius $R_c = \sqrt{\nu/m\omega_c}$ is supposed to be much more than the Bohr radius $a_B = 1/me^2$, namely, $R_c \gg a_B$.

2.2 Formalism

The system is described by the following grand canonical partition function in the path-integral representation

$$Z = \int \mathcal{D}[\overline{\psi}, \psi] \int \mathcal{D}[V_{dis}]\, \mathcal{P}[V_{dis}(\vec{r})]\, e^{S[\overline{\psi}, \psi, V_{dis}]} \tag{1}$$

$$\begin{aligned} S &= \sum_{\alpha=1}^{N_r} \int_0^{1/T} d\tau \int d^2\vec{r} \Big[\overline{\psi}^{\alpha,\sigma}(\vec{r},\tau) \left(-\partial_\tau + \mu - \mathcal{H} - V_{dis}(\vec{r})\right) \psi^{\alpha,\sigma}(\vec{r},\tau) \\ &- \frac{1}{2} \int d^2\vec{r}_1\, \overline{\psi}^{\alpha,\sigma}(\vec{r},\tau) \psi^{\alpha,\sigma}(\vec{r},\tau) U_0(\vec{r},\vec{r}_1) \overline{\psi}^{\alpha,\sigma_1}(\vec{r}_1,\tau) \psi^{\alpha,\sigma_1}(\vec{r}_1,\tau) \Big] \end{aligned} \tag{2}$$

Here $\psi^{\alpha,\sigma}$ and $\overline{\psi}^{\alpha,\sigma}$ are the Grassmann variables defined within the imaginary time interval $\tau \in [0, 1/T]$ with the antiperiodic condition $\psi(\vec{r}, 1/T) = -\psi(\vec{r}, 0)$. Symbol T stands for the temperature, μ is the chemical potential of the system, and $\sigma, \sigma_1 = \pm 1$ are spin indices. Hamiltonian $\mathcal{H} = (-i\nabla - e\vec{A})^2/(2m)$ describes an electron with mass m propagating in the two-dimensional space in the perpendicular magnetic field $H = \epsilon_{ab}\partial_a A_b$. The random potential $V_{dis}(\vec{r})$ is chosen to have the Gaussian distribution function

$$\mathcal{P}[V_{dis}(\vec{r})] = \sqrt{\rho\tau_0}\, \exp\left(-\pi\rho\tau_0 \int d^2\vec{r}\, V_{dis}^2(\vec{r}) \right) \tag{3}$$

where ρ denotes the thermodynamical density of states.

In order to find the average $\overline{\ln Z}$ over disorder, there are introduced N_r replicated copies of the system and $\alpha = 1, ..., N_r$ are replica indices.

The Matsubara representation seems to be more convenient for the problem concerned. Therefore the Fourier transform from imaginary time τ to the Matsubara frequencies will be employed. Since the fermionic fields are antiperiodic within the interval $[0, 1/T]$, the frequencies permitted for $\overline{\psi}$ and ψ are $\omega_n = \pi T(2n+1)$, n being an integer. The Fourier-transformed fields are defined, respectively

$$\overline{\psi}^{\alpha}(\tau) = \sqrt{T} \sum_{n=-\infty}^{\infty} \overline{\psi}_n^{\alpha} e^{i\omega_n \tau} \quad , \quad \psi^{\alpha}(\tau) = \sqrt{T} \sum_{n=-\infty}^{\infty} \psi_n^{\alpha} e^{-i\omega_n \tau} \tag{4}$$

Later on, the limits in the frequency and replica series will be omitted for a brevity.

In the Matsubara representation the action (2) becomes

$$\begin{aligned} S &= \int d^2\vec{r} \sum_{\alpha,n} \Big[\overline{\psi}_n^{\alpha,\sigma}(\vec{r}) \left(i\omega_n + \mu - \mathcal{H} - V_{dis}(\vec{r})\right) \psi_n^{\alpha,\sigma}(\vec{r}) \\ &- \frac{T}{2} \sum_{l,m} \int d^2\vec{r}_1\, \overline{\psi}_m^{\alpha,\sigma}(\vec{r}) \psi_{m-n}^{\alpha,\sigma}(\vec{r}) U_0(\vec{r},\vec{r}_1) \overline{\psi}_l^{\alpha,\sigma_1}(\vec{r}_1) \psi_{l+n}^{\alpha,\sigma_1}(\vec{r}_1) \Big] \end{aligned} \tag{5}$$

The Zeeman term in action (2) is neglected due to smallness of the g-factor. In fact, the condition $g \ll 1$ is usually hold. Nevertheless, the Zeeman term can be involved into the effective action after performing the integration over the fast degrees of freedom. To simplify the notation, the spin indices will be associated with the replica ones where it will be convenient.

2.3 Plasmon field and the average over disorder

The Coulomb term in action (5) is quartic in the fermionic fields. One can get rid of this quartic term by performing the Hubbard-Stratonovich transformation, introducing an extra path integration over bosonic field $\lambda_n^\alpha(\vec{r})$. With the help of the so-called plasmon field the Coulomb term can be presented as

$$\int \mathcal{D}[\lambda] \exp\left[-\frac{T}{2}\iint d^2\vec{r}\, d^2\vec{r}_1\, \lambda^\dagger(\vec{r}) U_0^{-1}(\vec{r},\vec{r}_1)\lambda(\vec{r}_1) + iT\int d^2\vec{r}\,\psi^\dagger(\vec{r})\hat{\lambda}(\vec{r})\psi(\vec{r})\right] \tag{6}$$

where U_0^{-1} stands for the operator inverse in U_0. The matrix notations are used for the combined replica and frequency indices

$$\psi^\dagger(\cdots)\psi = \sum_{n,m}^{\alpha,\beta} \overline{\psi}_n^\alpha (\cdots)_{nm}^{\alpha\beta}\psi_m^\beta \quad , \quad \lambda^\dagger\lambda = \sum_n^\alpha \lambda_{-n}^\alpha \lambda_n^\alpha \tag{7}$$

The quantities with the "hat" are defined according to $\hat{z} = \sum_{\alpha,n} z_n^\alpha I_n^\alpha$ with the matrix $(I_n^\alpha)_{kl}^{\beta\gamma} = \delta^{\alpha\beta}\delta^{\alpha\gamma}\delta_{k-l,n}$. The matrices I_n^α represent the diagonals shifted in the frequency space and, in general, are the generators of the $U(1)$-gauge transformations. The measure of the functional integral over the plasmon field λ is introduced so that integral (6) equals unity while fermionic fields $\psi^\dagger$ and ψ vanish.

In order to perform the averaging over disorder in the partition function (1), one should integrate over the random potential $V_{dis}(\vec{r})$. This leads straightforwardly to the following quartic term

$$\frac{1}{4\pi\rho\tau}\int d^2\vec{r}\sum_{nm}^{\alpha\beta} \overline{\psi}_n^\alpha(\vec{r})\psi_n^\alpha(\vec{r})\overline{\psi}_m^\beta(\vec{r})\psi_m^\beta(\vec{r}) \tag{8}$$

in the action. The term (8) can be decoupled by means of the Hubbard-Stratonovich transformation. An extra path integration over the Hermitian matrix field variables $Q_{nm}^{\alpha\beta}(\vec{r})$ can be introduced [11, 12]

$$\int \mathcal{D}[Q] \exp\int d^2\vec{r}\,\left[-\pi\rho\tau_0\,\mathrm{tr}\,Q^2(\vec{r}) + i\psi^\dagger(\vec{r})Q(\vec{r})\psi(\vec{r})\right] \tag{9}$$

Here the symbol tr denotes the matrix trace over the Matsubara, replica and spin spaces. The measure of the functional integral over the matrix field Q is defined in the same way as for the plasmon field, i.e., integral (9) equals unity while fermionic fields $\psi^\dagger$ and ψ vanish.

After calculations discussed above the partition function becomes

$$Z = \int \mathcal{D}[\overline{\psi},\psi,\lambda,Q]\, e^{S[\overline{\psi},\psi,\lambda,Q]} \tag{10}$$

$$\begin{aligned} S &= -\pi\rho\tau_0\int d^2\vec{r}\,\mathrm{tr}\,Q^2 - \frac{T}{2}\iint d^2\vec{r}\, d^2\vec{r}_1\,\lambda^\dagger(\vec{r})U_0^{-1}(\vec{r},\vec{r}_1)\lambda(\vec{r}_1) \\ &+ \int d^2\vec{r}\,\psi^\dagger(\vec{r})\left(i\omega + \mu - \hat{\mathcal{H}} + iT\hat{\lambda} + iQ\right)\psi(\vec{r}) \end{aligned} \tag{11}$$

Here ω is a unity matrix in the replica space while in the Matsubara space it is a matrix containing the frequencies ω_n on the diagonal, namely, $(\omega)_{nm}^{\alpha\beta} = \omega_n\delta_{nm}\delta^{\alpha\beta}$.

2.4 Elimination of the N-th Landau level

The fermionic fields $\psi^\dagger$ and ψ refer to all Landau levels. In order to integrate over all the fermionic degrees of freedom not belonging to the partially filled N-th Landau level, we separate the fermionic fields into two kinds. The first field refers to the N-th Landau level

$$\Psi(\vec{r}) = \sum_k \psi_{Nk}\varphi_{Nk}(\vec{r}) \quad , \quad \Psi^\dagger(\vec{r}) = \sum_k \psi_{Nk}^\dagger\varphi_{Nk}(\vec{r}) \tag{12}$$

The second one involves the other levels

$$\Phi(\vec{r}) = \sum_{p\neq N,k} \psi_{pk}\varphi_{pk}(\vec{r}) \quad , \quad \Phi^{\dagger}(\vec{r}) = \sum_{p\neq N,k} \psi^{\dagger}_{pk}\varphi_{pk}(\vec{r}) \tag{13}$$

where $\varphi_{pk}(\vec{r})$ are the eigenfunctions of the hamiltonian $\mathcal{H}$ and $p = 0, 1, ..., N, ...$ numerates the Landau levels with energies $\epsilon_p = \omega_c(p + 1/2)$. In addition, we introduce two types of the Green functions. One is for the N-th Landau level

$$G(\vec{r}, \vec{r}_1; Q, \lambda) = \sum_{k,k'} \varphi^{*}_{Nk}(\vec{r}) G_{Nk,Nk'}(Q, \lambda)\varphi_{Nk'}(\vec{r}_1) \tag{14}$$

and another is for the other levels

$$\tilde{G}(\vec{r}, \vec{r}_1; Q, \lambda) = \sum_{p,p'\neq N} \sum_{k,k'} \varphi^{*}_{pk}(\vec{r}) G_{pk,p'k'}(Q, \lambda)\varphi_{p'k'}(\vec{r}_1) \tag{15}$$

where the inverse of the Green function for the ψ_{pk} and $\psi^{\dagger}_{p'k'}$ operators is as follows

$$(G^{-1})_{pk,p'k'} = (i\omega + \mu - \epsilon_p)\delta_{pp'}\delta_{kk'} + iT\hat{\lambda}_{pk,p'k'} + iQ_{pk,p'k'} \tag{16}$$

with the following matrix elements

$$f_{pk,p'k'} = \int d^2\vec{r}\, \varphi^{*}_{p'k'}(\vec{r}) f(\vec{r}) \varphi_{pk}(\vec{r}) \tag{17}$$

The action (11) is bilinear in the fermionic fields $\psi^{\dagger}$ and ψ and obviously does in the fermionic fields $\Phi^{\dagger}$ and Φ too. Therefore one can integrate over the fermionic fields $\Phi^{\dagger}$ and Φ and obtain the following result

$$\begin{aligned} S = &- \int \mathrm{tr} \ln \tilde{G} - \pi\rho\tau_0 \int \mathrm{tr}\, Q^2 + \int \Psi^{\dagger} \left[i\omega + \mu - \hat{\mathcal{H}} + iT\hat{\lambda} + iQ \right] \Psi \\ - &\frac{T}{2} \iint \lambda^{\dagger} U_0^{-1} \lambda + \iint \left[\Psi^{\dagger} Q \tilde{G} Q \Psi + 2T\Psi^{\dagger}\hat{\lambda}\tilde{G}Q\Psi + T^2\Psi^{\dagger}\hat{\lambda}\tilde{G}\hat{\lambda}\Psi \right] \end{aligned} \tag{18}$$

Hereafter, the space indices are omitted. It should be noted that the last term in the action (18) appears due to the interaction between electrons belonging to the partially filled N-th Landau level and the other electrons.

2.5 Integration over the Q field

The Q matrix field should be divided into the transverse V and the longitudinal P components as follows $Q = V^{-1}PV$. Here the longitudinal component P has a block-diagonal structure in the Matsubara space, i.e., $P^{\alpha\beta}_{nm} \propto \Theta(nm)$ where $\Theta(x)$ is the Heaviside step function. The transverse component V corresponds to a unitary rotation. One can find a review on the above discussion in Refs. [13, 14].

The change of variables Q with P and V is motivated by the saddle-point structure of the action (18) in the absence of the plasmon field λ and at zero temperature, i.e., $\omega_n \to 0$. This saddle-point solution can be written as $Q_{sp} = V^{-1}P_{sp}V$ where the matrix P_{sp} obeys the equation

$$2\pi\rho\tau_0 P_{sp} = i\Big[G_0(\vec{r}, \vec{r}) + \tilde{G}_0(\vec{r}, \vec{r}) \Big] \tag{19}$$

which coincides with the self-consistent Born approximation equation [15]. Here the Green function G_0 is determined as a special case of G, namely, $G_0(\vec{r}, \vec{r}_1) = G(\vec{r}, \vec{r}_1; P_{sp}, 0)$ and the same for $\tilde{G}_0$.

In the case of small disorder $\omega_c\tau_0 \gg 1$ the solution of equation (19) has the form as

$$(P_{sp})^{\alpha\beta}_{nm} = \frac{\mathrm{sign}\, n}{2\tau}\delta_{nm}\delta^{\alpha\beta} \quad , \quad \tau = \pi\sqrt{\frac{\rho}{m}}\frac{\tau_0}{\sqrt{\omega_c\tau_0}} \tag{20}$$

The presence of the plasmon field λ results in a shift of the saddle-point value (20) of the P field which can be found by expanding the action (18) to second order in both λ and $\delta P = P - P_{sp}$. Hence one can obtain

$$S = S_0 + S_1[\delta P, \lambda] + S_2[\overline{\Psi}, \Psi, \delta P, \lambda] \tag{21}$$

$$S_0 = \int \left(\operatorname{tr} \ln \tilde{G}_0 - \pi\rho\tau_0 \operatorname{tr} Q_{sp}^2 + \Psi^\dagger \left[i\omega + \mu - \hat{\mathcal{H}} + iT\hat{\lambda} + iQ_{sp} \right] \Psi \right) \tag{22}$$

$$S_1 = iT \int \operatorname{tr} \tilde{G}_0 \hat{\lambda} - \pi\rho\tau_0 \int \operatorname{tr}(\delta P)^2 - \frac{T}{2} \iint \lambda^\dagger U_0^{-1} \lambda - \frac{1}{2} \iint \operatorname{tr} \left[T\hat{\lambda} + \delta P \right] \pi_0 \left[T\hat{\lambda} + \delta P \right] \tag{23}$$

$$S_2 = \iint \left(\Psi^\dagger \left[T\hat{\lambda} + \delta P \right] \tilde{G}_0 \left[T\hat{\lambda} + \delta P \right] \Psi - 2 \operatorname{tr} \left[T\hat{\lambda} + \delta P \right] \tilde{G}_0 \left[T\hat{\lambda} + \delta P \right] G_0 \right) \tag{24}$$

where a bare polarization operator π_0 is implied to be a matrix according to the rule

$$\operatorname{tr} A\pi_0 B = \sum_{nm}^{\alpha\beta} A_{m+n,m}^{\alpha\beta}(\vec{r}) \pi_0^m(n; \vec{r}, \vec{r}_1) B_{m,m+n}^{\beta\alpha}(\vec{r}_1) \tag{25}$$

and is defined by

$$\pi_0^m(n; \vec{r}, \vec{r}_1) = -2 \left(\tilde{G}_0^{m+n}(\vec{r}, \vec{r}_1) \tilde{G}_0^m(\vec{r}_1, \vec{r}) + \tilde{G}_0^{m+n}(\vec{r}, \vec{r}_1) G_0^m(\vec{r}_1, \vec{r}) + G_0^{m+n}(\vec{r}, \vec{r}_1) \tilde{G}_0^m(\vec{r}_1, \vec{r}) \right) \tag{26}$$

After decomposing of the matrix field Q into the block-diagonal Hermitian matrix field P and unitary matrix field V, the measure of the functional integral in (21) becomes $\mathcal{D}[Q] = \mathcal{D}[V]\mathcal{D}[\delta P] I[\delta P]$ where [13]

$$\ln I[\delta P] = -\frac{1}{(\pi\rho)^2} \int \sum_{nm}^{\alpha\beta} \left[1 - \Theta(nm) \right] \delta P_{nn}^{\alpha\alpha} \delta P_{mm}^{\beta\beta} \tag{27}$$

The terms which are quadratic in δP from the term S_1 of action (21) together with the contribution from measure (27) determine the propagator of the δP fields

$$\begin{aligned} \langle \delta P_{m_1 m_2}^{\alpha\beta}(\vec{q}) \delta P_{m_3 m_4}^{\gamma\delta}(-\vec{q}) \rangle &= \frac{\delta_{m_1 m_4} \delta_{m_2 m_3} \delta^{\alpha\delta} \delta^{\beta\gamma} \dfrac{\Theta(m_1 m_3)}{2\pi\rho\tau_0}}{1 + \dfrac{\pi_0^{m_1}(m_3 - m_1; \vec{q})}{2\pi\rho\tau_0}} \\ &- \frac{2\left[1 - \Theta(m_1 m_3)\right]}{(2\pi^2\rho^2\tau)^2} \frac{\delta_{m_1 m_2} \delta^{\alpha\beta}}{1 + \dfrac{\pi_0^{m_1}(0; \vec{q})}{2\pi\rho\tau_0}} \frac{\delta_{m_3 m_4} \delta^{\delta\gamma}}{1 + \dfrac{\pi_0^{m_3}(0; \vec{q})}{2\pi\rho\tau_0}} \end{aligned} \tag{28}$$

It should be noted that the propagator of the longitudinal fluctuations (28) proves to be analogous to that previously obtained for the problem on the behavior of a free electron gas in the perpendicular magnetic field [13].

Using the expression (28) for the propagator of the δP fields, one can integrate action (21) over the longitudinal fluctuations in the quadratic approximation. This yields the following result

$$S = S_0 + S_\lambda + S_\mu \tag{29}$$

where S_0 given by equation (22) describes the electrons at the partially filled N-th Landau level coupled to the plasmon and Q_{sp} fields. The term S_λ concerns the screening of the Coulomb interaction due to the influence of electrons from the other Landau levels and is given by

$$S_\lambda = iT \int d^2\vec{r} \operatorname{tr} \tilde{G}_0(\vec{r}, \vec{r}) \hat{\lambda}(\vec{r}) - \frac{T}{2} \int \frac{d^2\vec{q}}{(2\pi)^2} \sum_n^\alpha \lambda_{-n}^\alpha(\vec{q}) U_0^{-1}(q) \varepsilon(n, \vec{q}) \lambda_n^\alpha(-\vec{q}) \tag{30}$$

where the dielectric function $\varepsilon(n,q) = 1 + U_0(q)\Pi(n,q)$ with the following polarization operator [1]

$$\begin{aligned}\Pi(n,q) &= T\sum_m \pi_0^m(n,q)\left[1 - \frac{\Theta(n(n+m))\dfrac{\pi_0^m(n,q)}{2\pi\rho\tau_0}}{1+\dfrac{\pi_0^m(n,q)}{2\pi\rho\tau_0}}\right] \\ &+ T\frac{\delta_{n,0}}{(\pi^2\rho^2\tau)^2}\sum_{km}\frac{[1-\Theta(km)]\pi_0^m(0,q)}{1+\dfrac{\pi_0^m(0,q)}{2\pi\rho\tau_0}}\frac{\pi_0^k(0,q)}{1+\dfrac{\pi_0^k(0,q)}{2\pi\rho\tau_0}} \end{aligned} \tag{31}$$

The third term S_μ of action (29) contains the terms which affect the chemical and thermodynamic potentials of the system (See Appendix A).

It is worthwhile to mention that the saddle-point approximation in which the integration over the Q field is performed is valid since the condition $\mu\tau = N\omega_c\tau \gg 1$ is hold.

2.6 Integration over the plasmon field

As a final step of the procedure, the action (29) should be integrated over the plasmon field λ. The integration can be performed in the quadratic approximation in the λ fields. The corresponding propagator is determined by the second term in Eq.(30). After that one obtains the effective action for electrons on the partially filled Landau level which is the main result of the paper

$$\begin{aligned} S_{eff} &= -\frac{\Omega}{T} + \int d^2\vec{r}\,\Psi^\dagger(\vec{r})\left[i\omega + \tilde{\mu} - \hat{\mathcal{H}} + iQ_{sp}\right]\Psi(\vec{r}) - \pi\rho\tau_0\int d^2\vec{r}\,\mathrm{tr}\,Q_{sp}^2(\vec{r}) \\ &- \frac{T}{2}\iint d^2\vec{r}\,d^2\vec{r}_1\sum_{nmk}^{\alpha}\overline{\Psi}_m^{\alpha,\sigma}(\vec{r})\Psi_{m+n}^{\alpha,\sigma}(\vec{r})U_{eff}(\vec{r}-\vec{r}_1)\overline{\Psi}_k^{\alpha,\sigma_1}(\vec{r}_1)\Psi_{k-n}^{\alpha,\sigma_1}(\vec{r}_1) \\ &+ \frac{g\omega_c}{2}\int d^2\vec{r}\sum_n^{\alpha}\sigma\overline{\Psi}_n^{\alpha,\sigma}(\vec{r})\Psi_n^{\alpha,\sigma}(\vec{r}) \end{aligned} \tag{32}$$

where we incorporate the Zeeman term into the effective action. The Fourier transformation of the effective interaction potential $U_{eff}(q) = U_0(q)/\varepsilon(q)$ is determined by the static dielectric function $\varepsilon(q) \equiv \varepsilon(0,q)$. In general, the low energy properties of the system concerned can be described with the help of the retarded interaction alone (see action (29)). However, the description within the framework of the effective action with the instantaneous interaction seems to be a rather good approximation in the problem under consideration [7]. This is due to the fact that the transitions between the Landau levels have a characteristic time scale about ω_c^{-1} while the typical energy scale in the effective theory is of the order of the exchange energy $\Delta_{ex} \ll \omega_c$(see Sec. 4).

The existence of the other Landau level with the exceptions of the partially filled N-th one produces an effect on both the thermodynamic and the chemical potentials. The thermodynamic potential Ω in action (32) can be presented in the following way

$$\Omega = \Omega_0 + \Delta\Omega \tag{33}$$

where Ω_0 is the thermodynamic potential of the system of noninteracting electrons for the completely filled Landau levels in the presence of disorder

$$\Omega_0 = T\int d^2\vec{r}\,\mathrm{tr}\ln\tilde{G}_0(\vec{r},\vec{r}) \tag{34}$$

and the quantity $\Delta\Omega$ is analogous to first-order exchange and correlation correction equivalent to the sum of ring diagrams and contributing to the ground state energy of a clean electron liquid [16]

$$\Delta\Omega = -\frac{T}{2}\int d^2\vec{r}\sum_n\int\frac{d^2\vec{q}}{(2\pi)^2}\ln\varepsilon(n,\vec{q}) \tag{35}$$

[1] It should be noted that similar form of the polarization operator but with another bare polarization operator $\pi_0^m(n,q)$ was first derived by Baranov and Pruisken [17]

The chemical potential $\tilde{\mu}$ in action (32) can be written as

$$\tilde{\mu} = \mu + \delta\mu \tag{36}$$

where the shift of the chemical potential

$$\delta\mu = 2\pi l^2 T \sum_n \int d^2\vec{r}\, \tilde{G}_0^n(0,\vec{r}) P_N(0,\vec{r}) U_{eff}(n,\vec{r}) \tag{37}$$

contains the corrections similar to the exchange and correlation ones in a clean electron liquid. Here $l = 1/\sqrt{m\omega_c}$ is the magnetic field length. The quantity $U_{eff}(n,\vec{r})$ is a Fourier transform of $U_0(\vec{q})/\varepsilon(n,\vec{q})$ and

$$P_N(\vec{r}_1,\vec{r}_2) = \sum_k \varphi^*_{Nk}(\vec{r}_2)\varphi_{Nk}(\vec{r}_1) \tag{38}$$

is the projectional operator onto the partially filled N-th Landau level.

It should be noted that the corrections to the thermodynamical and chemical potentials contain additional terms except ones presented above. They are neglected in the limit of a weak disorder $\omega_c\tau \gg 1$ (see Appendix A).

The integration over the plasmon field is performed in the Gaussian approximation. It can be justified provided the fluctuations of the plasmon field are small. The long and short-range fluctuations are different physically. In the case of large length scale $r \gg R_c$, the dipole transitions between the adjacent Landau levels are induced only. The long-range fluctuations are small if the condition $Nr_s \gg 1$ satisfies [7]. Physically, this condition means that the characteristic length scale R_c^2/a_B of the long-range fluctuations should be much more then the cyclotron radius R_c. The short-range fluctuations correspond to the case of small length scale $r \ll R_c$. The transitions between distant Landau levels are possible in this case. The condition $r_s \ll 1$ of smallness for the short-range fluctuations is just the criterion of perturbation theory applicability to the Coulomb interaction.

3 Effective interaction, the thermodynamic and chemical potentials

The results of previous section allow to find the effective action (32) for the electrons on the partially filled N-th Landau level. The main physical quantity which affects the dynamics of the electrons is the effective electron-electron interaction. It is completely determined by the static dielectric function $\varepsilon(q)$. The other two interesting quantities in the effective action (32) are the thermodynamical and chemical potentials.

3.1 Effective interaction

The most pronounced effect of electrons on the completely filled Landau levels is the screening of the electron-electron interaction on the partially filled Landau level. This screening is determined by the static dielectric function $\varepsilon(q)$.

According to the equation (31) for the polarization operator $\Pi(n,q)$, the dielectric function can be obtained for arbitrary value of the disorder parameter $\omega_c\tau$. However the situation of the small broadening of Landau level due to the disorder is the most interest from the physical point of view. In this case the expression for the static dielectric function can be simplified drastically

$$\varepsilon(q) = 1 + \frac{2\pi e^2}{q} T \sum_n \pi_0^n(0,q)\,, \qquad \omega_c\tau \gg 1 \tag{39}$$

The evaluation of the static dielectric function is presented in Appendix B. The result of evaluation can be written as

$$\varepsilon(q) = 1 + \frac{2}{qa_B}\left(1 - \frac{\pi}{6\tau\omega_c}\right)\left(1 - \mathcal{J}_0^2(qR_c)\right) \tag{40}$$

Here $\mathcal{J}_0(x)$ is the Bessel function of the first kind. Expression (40) for the static dielectric function is the main result of the paper.

It is worthwhile to note that the asymptotic expressions (in the $qR_c \ll 1$ and $qR_c \gg 1$ domains) for the static dielectric function $\varepsilon(q)$ in a clean system ($\tau^{-1} = 0$) was obtained earlier by Kukushkin, Meshkov and Timofeev [18]. The general expression for the static dielectric function in a clean system was derived by Aleiner and Glazman [7].

We mention that the asymptotic expressions for the static dielectric function in a clean system can be obtained from a clear physical picture [18, 7]. The behavior of the static dielectric function in the region $qR_c \ll 1$ can be explained by dipole transitions between the adjacent Landau levels. The result for the static dielectric function in the region $qR_c \gg 1$ is explained by the standard Thomas-Fermi screening. But there is no clear physical picture in the case of a weak dirty system. We have no other opportunity to obtain the dielectric function except the derivation of the effective action for electrons on the partially filled Landau level.

According to equation (40) in the $qR_c \ll 1$ domain the static dielectric function is as follows

$$\varepsilon(q) = 1 + \Big(1 - \frac{\pi}{6\omega_c\tau}\Big)\frac{R_c^2 q}{a_B} \tag{41}$$

The above result shows that the disorder suppresses the effect of the screening. We can expect that while the disorder increases the screening decreases. We can estimate the disorder threshold τ^*, i.e., the point of vanishing screening, as $\omega_c\tau^* \sim \pi/6$.

One can obtain from equation (40) the following expression for the static dielectric function in the $qR_c \gg 1$ domain

$$\varepsilon(q) = 1 + \frac{2}{qa_B}\Big(1 - \frac{\pi}{6\omega_c\tau}\Big)\Big(1 - \frac{1 + \sin 2qR_c)}{\pi qR_c}\Big) \tag{42}$$

The disorder suppresses the screening also in the region of large wave vectors $qR_c \gg 1$.

Equations (40) allow us to obtain the asymptotic behavior of the effective interaction $U_{eff}(r)$ in the coordinate space. The polarization is insignificant for the very large length scale $r \gg R_c^2/a_B$ and the effective interaction coincides with the bare Coulomb interaction

$$U_{eff}(r) = \frac{e^2}{r}\left(1 - \frac{R_c^4}{a_B^2 r^2}\left[1 - \frac{\pi}{3\omega_c\tau}\right]\right) \tag{43}$$

At the intermediate scale $R_c^2/a_B \gg r \gg R_c$ the polarization becomes important and the effective interaction reads

$$U_{eff}(r) = \frac{\omega_c}{2N\Big(1 - \frac{\pi}{6\omega_c\tau}\Big)}\ln\Big(1 + \frac{R_c^2(1 - \frac{\pi}{6\omega_c\tau})}{a_B r}\Big) \tag{44}$$

We note that while disorder increases the effective interaction tends to the bare Coulomb interaction. For the small scale $R_c \gg r \gg a_B$ the Thomas-Fermi screening takes place and the effective interaction has the following form

$$U_{eff}(r) = \frac{e^2 a_B^2}{r^3}\Big(1 - \frac{\pi}{6\omega_c\tau}\Big) \tag{45}$$

It should be emphasized that the disorder in the system affects the electron-electron most strongly within the intermediate length scale $R_c^2/a_B \gg r \gg R_c$. Physically, this is the case in which dipole transitions between the adjacent Landau levels are possible.

3.2 The thermodynamic and chemical potentials

The thermodynamic and chemical potentials (34-37) can be evaluated in the leading orders in $1/N$. The detailed calculations are presented in Appendix C.

The thermodynamic potential for the system of noninteracting electrons in the presence of disorder for the completely filled Landau levels is given by

$$\Omega_0 = \frac{L_x L_y}{\pi l^2}\left[\frac{N(N-1)}{2}\omega_c - \mu - \frac{\ln(2\omega_c\tau)-1}{\pi\tau}\right] \tag{46}$$

where L_x and L_y are the sizes of the system. The first-order exchange correction to the thermodynamical potential reads

$$\Delta\Omega = -\frac{L_x L_y}{\pi l^2}\frac{e^2}{\pi l}(2N)^{3/2}\left[\frac{2}{3} + \frac{2\ln 2}{\pi\omega_c\tau}\frac{1}{2N}\right] \tag{47}$$

The presence of disorder changes the dependence of $\Delta\Omega$ on the magnetic field, i.e., N. For the dirty system, the second term in brackets of equation (47) is proportional to $1/N$. This is in contrast to the clean system where the correction is much smaller and proportional to $1/N^2$ [7].

The shift of the chemical potential due to exchange correction can be written as

$$\delta\mu = \frac{2e^2}{\pi l}(2N)^{1/2}\left[1 - \frac{\ln N}{8N} + \frac{1}{\pi\omega_c\tau}\frac{1}{2N}\right] \tag{48}$$

It should be noted that $\delta\mu$ contains only the exchange correction and does not include the correlation correction due to normal ordering of the $\Psi^\dagger$ and Ψ fields (see Ref. [7]).

4 Spin excitations

In the previous section we analyzed the renormalization of the electron-electron interaction on the partially filled N-th Landau level due to the existence of the other levels. In this section we investigate the enhancement of the g-factor and the simplest spin excitations at the filling factor $\nu = 2N+1$.

The electrons on the partially filled N-th Landau level at the filling factor $\nu = 2N+1$ possess a maximal spin in the ground state, since the ground state does not contain skyrmions at large ν [22]. This ground state is obviously fully spin-polarized and described by the following wave function $|N_{el} = N_\Phi, S_z = N_\Phi/2\rangle$ where N_{el} is the number of electrons on the partially filled N-th Landau level and $N_\Phi = L_x L_y/(2\pi l^2)$ is the number of states on the Landau level. The simplest excitations are described by the state of energy $E_\uparrow$ with an extra hole and the state of energy $E_\downarrow$ with an extra electron. The width of the spin gap Δ_s is related with the energies of the excited states and with the energy E_0 of the ground state [19, 20, 7] as follows $\Delta_s = E_\uparrow + E_\downarrow - 2E_0$. We can obtain that the width of the spin gap equals $\Delta_s = \Delta_{ex} + g\omega_c$ where the shift of the chemical potential Δ_{ex} due to the exchange interaction [23, 20] is determined by

$$\Delta_{ex} = 2\pi l^2 \int d^2\vec{r}\, U_{eff}(\vec{r}) P_N(0,\vec{r}) P_N(\vec{r},0) \tag{49}$$

Using expression (38) for the projectional operator P_N we can evaluate the effective g-factor. It is defined as $g_{eff} = \Delta_s/\omega_c$ and reads

$$g_{eff} = g + \frac{r_s}{\pi\sqrt{2}}\ln\Big[\frac{2\sqrt{2}}{r_s}(1 - \frac{\pi}{6\omega_c\tau})\Big] + \frac{E_h}{\omega_c} \tag{50}$$

where a "hydrodynamic" term is

$$E_h = \frac{\omega_c}{2N\Big(1 - \dfrac{\pi}{6\omega_c\tau}\Big)}\ln\Big[1 + \sqrt{2}r_s N(1 - \frac{\pi}{6\omega_c\tau})\Big)\Big] \tag{51}$$

The disorder in the system results in the enhancement of the effective g-factor.

Now we are in the position to discuss the neutral excitations — spin waves [20, 24] at the filling factor $\nu = 2N+1$. They are described by the following wave function

$$\sum_q e^{ik_x q l^2}\overline{\Psi}_{N,q,\downarrow}\Psi_{N,q-k_y,\uparrow}|N_\Phi, \frac{N_\Phi}{2}\rangle \tag{52}$$

Following Ref. [20], we should take into account three contributions. They are the difference of the exchange self-energy of an electron in the excited Landau level and the self-energy in the level from which the electron removed, the direct Coulomb interaction and the exchange energy. Then we obtain the equation which determines the spectrum of the spin wave excitations

$$\omega = g\omega_c + \int \frac{d^2\vec{q}}{(2\pi)^2}\frac{U_0(q)}{\varepsilon(q,\omega)}\left[L_N\left(\frac{q^2l^2}{2}\right)\right]^2 e^{-q^2l^2/2}\Big(1 - e^{i\vec{k}\vec{q}l^2}\Big) \tag{53}$$

where $L_N(x)$ is the Laguerre polynomial. The dielectric function $\varepsilon(q,\omega)$ contains the imaginary part (see Eq.(B.4)) which is of order of $1/\omega_c\tau$. It results in the decay rate of the spin wave excitations. Physically, the spin wave excitations decay due to the scattering on impurities. We mention that the decay rate appears also in the magnetoplasmon spectrum.

The energy of the spin wave excitations is much less then ω_c, $\omega(k) \ll \omega_c$. Thus we can calculate the real $E_{SW}(k)$ and imaginary $\Gamma_{SW}(k)$ part of the spin-wave energy separately. We put ω to zero in the right hand side of Eq.(53). Then the evaluation of equation (53) leads to a quadratic dispersion relation for the small wave vectors $kR_c \ll 1$

$$E_{SW}(k) = g\omega_c + \frac{r_s\omega_c}{\pi\sqrt{2}}\Big[1 + \frac{r_s}{\sqrt{2}}(1 - \frac{\pi}{6\omega_c\tau})\Big]^{-1}(kR_c)^2 \tag{54}$$

The disorder suppresses the effective mass of the spin wave excitations. For the sufficiently large wave vectors $1 \ll kR_c \ll R_c^2/l^2$, the energy of spin wave is given by

$$\begin{aligned} E_{SW}(k) &= \Delta_{ex} - E_h - \frac{r_s\omega_c}{\pi\sqrt{2}}\Bigg[\ln\Big(1 + \frac{(\sqrt{2}r_skR_c)^{-1}}{1 - \dfrac{\pi}{6\omega_c\tau}}\Big) \\ &+ \frac{\sin 2kR_c}{2kR_c}\Big(1 + \frac{r_s}{\sqrt{2}}(1 - \frac{\pi}{6\omega_c\tau})\Big)\Bigg] \end{aligned} \tag{55}$$

In order to obtain the decay rate of the spin wave excitations we take into account that the imaginary part $\varepsilon^{''}$ of the dielectric function is small. Then we obtain

$$\Gamma_{SW}(k) = -\int \frac{d^2\vec{q}}{(2\pi)^2}\frac{U_0(q)\varepsilon^{''}(q,E_{SW})}{\varepsilon_0^2(q,E_{SW})}\left[L_N\left(\frac{q^2l^2}{2}\right)\right]^2 e^{-q^2l^2/2}\Big(1 - e^{i\vec{k}\vec{q}l^2}\Big) \tag{56}$$

The evaluation of equation (56) for the small wave vectors $kR_c \ll 1$ results in

$$\Gamma_{SW} = -\frac{\arctan(2\omega_c\tau g)}{6\omega_c\tau}\frac{e^2}{a_B}(kR_c)^2\frac{1}{(1 + \dfrac{l^2}{a_BR_c})^2}\frac{2 - \sin 4N}{(4N)^2} \tag{57}$$

and for the large wave vectors $kR_c \gg 1$

$$\Gamma_{SW} = -\frac{\arctan(2\omega_c\tau g_{eff})}{\pi\omega_c\tau}\frac{e^2}{a_B}\Big[\Big(\frac{a_B}{R_c}\Big)^2\ln\frac{R_c}{a_B} + \frac{\mathrm{arccosh}(2kR_c)}{2(4N)^2}\Big] \tag{58}$$

We note that the decay rate Γ_{SW} is the same order as the corrections to the real part of the spin-wave energy E_{SW} due to the presence of disorder.

5 Zero-bias anomaly

In this section we consider an electron tunneling into a two-dimensional electron liquid with disorder in a weak magnetic field. We investigate suppression of the tunneling conductivity near zero bias, the so-called zero-bias anomaly. The properties of an electron tunneling into the electron system are usually described by dependence of the tunneling conductivity $G(V)$ on bias V. Recently, the effective action approach for the

zero-bias problem was developed by Levitov and Shytov [25]. The effective action describes spreading of a tunneling electron within the electron system in imaginary time ζ.

Following Ref. [25], the action of a spreading charge for the case of zero bias $V = 0$ is determined by

$$S_0(\zeta) = 4 \int_0^{+\infty} \frac{d\omega}{2\pi} \int_0^{+\infty} \frac{q\, dq}{2\pi} \frac{\sin^2 \omega\zeta}{\omega + Dq^2} \frac{U_{eff}(q)}{\omega + Dq^2 + \sigma q^2 U_{eff}(q)} \tag{59}$$

where σ and D are conductivity and diffusive constant of the electron system, respectively. They relate to each other by the Einstein's formula $\sigma = e^2 \rho D$.

Using asymptotic expression (41) for the static dielectric function $\varepsilon(q)$ we evaluate the action (59) in the large time $\zeta \gg 1$ limit

$$S_0(\zeta) = \frac{e^2}{8\pi^2 \sigma \eta} \ln \frac{2\zeta}{\tau_0} \ln \left(\frac{2\zeta}{\tau_0} \beta^{4\eta} \right) \tag{60}$$

where we introduce two dimensionless parameters

$$\beta = \frac{a_B}{\sqrt{2} l_{el}} \qquad , \qquad \eta = \left(1 - \frac{\pi}{6\omega_c \tau} \right) \left(\frac{R_c}{\sqrt{2} l_{el}} \right)^2 \tag{61}$$

with the bare elastic mean free path $l_{el} = R_c \omega_c \tau_0$. According to the inequality $a_B \ll R_c \ll l_{el}$, the parameters β and η are small, namely, $\beta \ll 1$ and $\eta \ll 1$.

Taking into account the work done by voltage source, we obtain the total action of the spreading charge $S(\zeta) = S_0(\zeta) - 2eV\zeta$. Then we should find an optimal time ζ_* which is determined by the minimum of the action $S(\zeta)$. The optimal time ζ_* plays the role of the charge accommodation time in the problem. It can be written as

$$\zeta_* = \tau_0 \frac{V_0}{2V} \ln \frac{V_0}{\beta^{2\eta} V} \qquad , \qquad eV_0 = \left(1 - \frac{\pi}{6\omega_c \tau} \right)^{-1} \frac{1}{\pi m R_c^2} \tag{62}$$

The theory should be self-consistent in the hydrodynamics limit, i.e., $\zeta_* \geq \tau_0$. Hence the theory is fulfilled for bias $V \leq V_0$.

Assuming the contribution from the barrier being just a constant at small bias, we can write the tunneling conductivity as follows

$$G(V) = G_0 \exp[-S_0(\zeta_*) + 2eV\zeta_*] \tag{63}$$

After performing evaluation we obtain the dependence of the tunneling conductivity on small bias

$$G(V) = G_0 \left(\frac{V}{V_0} \right)^{\alpha(V)} \qquad , \qquad \alpha(V) = \frac{e^2}{8\pi^2 \sigma \eta} \ln \frac{V_0}{V \beta^{4\eta}} \tag{64}$$

Equation (64) shows that the screening of the electron-electron interaction results in increasing of the suppression of the tunneling conductivity. We mention that the above result is valid for the bias being in the following range $V \leq V_0$.

The expression (64) for the tunneling conductivity contains the energy scale eV_0 which coincides with the "hydrodynamic" term E_h in Eq.(51) except the logarithm. A hydrodynamic model for the low-energy excitations of a clean ($\tau^{-1} = 0$) electron liquid in a weak magnetic field was considered by Aleiner, Baranger and Glazman [6]. They showed that the tunneling density of states exhibits a gap $2E_h$ tied to the Fermi energy. Equation (51) describes the same gap in the case of a weak disorder $\omega_c \tau \gg 1$. Apparently, it is disorder to be responsible for the fact that the gap is about $0.05\omega_c$ in a wide range of the applied magnetic field [4].

As magnetic field increases the factor α increases and becomes of order of unity. The zero-bias anomaly in the tunneling conductivity crossovers from weak to strong. The expression (64) shows that the factor α

depends on bias V and magnetic field. It results in the shift alone bias V of the crossover point V_c with the change of the applied magnetic field

$$V_c = V_0 \exp\left(-\frac{4\pi\mu}{\omega_c^2\tau_0}\right) \tag{65}$$

where μ is the chemical potential. The crossover was observed by Ashoori et al. [4] in the tunneling current from a normal metal into a two-dimensional electrons in the presence of a magnetic field. In the experiment the ohmic conductance was measured as a function of temperature T. For low temperatures the conductance corresponds to the zero temperature conductance taken at $V = T/e$. The two-dimensional electrons were relatively clean with the elastic collision time $\tau_0 \approx 4 \cdot 10^{-12}s$. The chemical potential calculated from the electron density was $\mu = 10mV$. Using Eq.(65) the dependence of crossover temperature on magnetic field can be written as

$$T_c = 2.9 \exp\left(-\left[\frac{3.2}{H}\right]^2\right) \tag{66}$$

where temperature is measured in *Kelvin* while magnetic field in *Tesla*. Eq.(66) provides a good agreement with the results reported in Ref. [4].

6 Conclusion

We considered the system of a two-dimensional electron gas in the presence of both the disorder and the Coulomb interaction in a weak perpendicular magnetic field. The effective low energy theory describing electrons at the partially filled N-th Landau level was derived in the case of a weak magnetic field ($Nr_s \gg 1$) and a weak interaction ($r_s \ll 1$). The modified electron-electron interaction for electrons on the partially filled Landau level takes into account the screening from the other electrons on the occupied Landau levels. We also presented the exchange corrections to the thermodynamical and chemical potentials in the presence of disorder.

The theory proposed allows us to take into account the effects of disorder in the problems connected with the behavior of a two-dimensional electron gas in the weak magnetic field. How disorder affects the formation of stripes, bubble phase [26], tunneling density of states, spin excitations, tunneling conductivity and so on can be investigated.

We discussed the effect of disorder on the exchange enhancement of the g-factor and the simplest spin excitations on the partially filled Landau level. We obtained an additional dependence of the effective g-factor as a function of the magnetic field, the suppression of the effective mass and the decay rate of the spin wave excitations.

We also investigated an electron tunneling into a two-dimensional electron liquid with a weak disorder in a weak magnetic field. We obtained the enhancement of the gap in the tunneling density of states and nonlinear dependence of tunneling conductivity on the applied bias.

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8 Appendix A

In this appendix the calculation of the term S_μ in action (29) is presented. This term appears after performing the integration over the longitudinal fluctuations and equals

$$S_\mu = \frac{1}{2}\langle\left(S_2[\overline{\Psi}, \Psi, \delta P, \lambda]\right)^2\rangle_{\delta P} \tag{A.1}$$

where $\langle\cdots\rangle_{\delta P}$ means the average with the propagator of the δP field (28). One can obtain

$$S_\mu = \delta S_1 + \delta S_2 + \delta S_3 \tag{A.2}$$

where

$$\delta S_1 = T^2 \iint d^2\vec{r}_1\, d^2\vec{r}_2 \left(\Psi^\dagger \hat{\lambda} \tilde{G}_0 \hat{\lambda} \Psi - 2\,\mathrm{tr}\, \hat{\lambda} \tilde{G}_0 \hat{\lambda} G_0 \right) \tag{A.3}$$

$$\delta S_2 = -T^2 \int d^2\vec{r}_1 \cdots \int d^2\vec{r}_4 \sum_{k,n,m}^{\alpha\beta} \lambda_m^\beta(\vec{r}_1) D_{nm}^{\alpha\alpha}(k;\vec{r}_3,\vec{r}_4) \pi_0^n(m;\vec{r}_1,\vec{r}_2) L_{n,n+m}^{\alpha\beta,\alpha\alpha}(\vec{r}_2,\vec{r}_4) \tag{A.4}$$

$$\delta S_3 = \frac{T^2}{2} \int d^2\vec{r}_1 \cdots \int d^2\vec{r}_4 \sum_{k,l,n,m}^{\alpha\beta\gamma\delta} D_{nm}^{\alpha\beta}(k;\vec{r}_1,\vec{r}_2) D_{nm}^{\gamma\delta}(l;\vec{r}_3,\vec{r}_4) L_{n,n+m}^{\alpha\beta,\gamma\delta}(\vec{r}_2,\vec{r}_4) \tag{A.5}$$

Here

$$\begin{aligned} & D_{nm}^{\alpha\beta}(k;\vec{r}_1,\vec{r}_2) \\ = & \left(\overline{\Psi}_k^\alpha(\vec{r}_1) \Psi_{n+m}^\beta(\vec{r}_2) - 2\delta_{k,n+m}\delta^{\alpha\beta} G_0^{n+m}(\vec{r}_2,\vec{r}_1) \right) \lambda_{n-k}^\alpha(\vec{r}_1) \tilde{G}_0^n(\vec{r}_1,\vec{r}_2) \\ + & \left(\overline{\Psi}_n^\beta(\vec{r}_1) \Psi_k^\alpha(\vec{r}_2) - 2\delta_{k,n}\delta^{\alpha\beta} G_0^n(\vec{r}_2,\vec{r}_1) \right) \lambda_{k-n-m}^\beta(\vec{r}_1) \tilde{G}_0^{n+m}(\vec{r}_1,\vec{r}_2) \end{aligned} \tag{A.6}$$

and $L_{m_1 m_2}^{\alpha\beta,\gamma\delta}$ is the propagator of the longitudinal fluctuations (28).

Performing the integration over the plasmon field, one obtains from the S_μ term

$$S_\mu \to \frac{\Delta\Omega_1 + \Delta\Omega_2 + \Delta\Omega_3}{T} + (\delta\mu_1 + \delta\mu_2 + \delta\mu_3) \int d^2\vec{r}\, \Psi^\dagger(\vec{r}) \Psi(\vec{r}) \tag{A.7}$$

where second-order corrections to the thermodynamic potential are given by

$$\begin{aligned} \frac{\Delta\Omega_1}{L_x L_y} &= -T^2 \sum_{nm} \int d^2\vec{r}\, G_0^n(0,\vec{r}) \tilde{G}_0^m(\vec{r},0) U_{eff}(m-n,\vec{r}) \\ \frac{\Delta\Omega_2}{L_x L_y} &= 2T^2 \sum_{nm} \int d^2\vec{r}\, d^2\vec{r}_1\, d^2\vec{r}_2 \left(G_0^n(0,\vec{r}) \tilde{G}_0^{n+m}(\vec{r},0) + \tilde{G}_0^n(\vec{r},0) G_0^{n+m}(0,\vec{r}) \right) \\ &\times U_{eff}(m,\vec{r}_1) \pi_0^n(m,\vec{r}_1-\vec{r}_2) L_{n,n+m}^{\alpha\alpha,\alpha\alpha}(\vec{r}_2-\vec{r}) \\ \frac{\Delta\Omega_3}{L_x L_y} &= 2T^2 \sum_{nm} \int d^2\vec{r}\, d^2\vec{r}_1\, d^2\vec{r}_2 \left(G_0^n(0,\vec{r}) \tilde{G}_0^{n+m}(\vec{r},0) + G_0^{n+m}(0,\vec{r}) \tilde{G}_0^n(\vec{r},0) \right) \\ &\times \left(G_0^n(0,\vec{r}_1) \tilde{G}_0^{n+m}(\vec{r}_1,0) + G_0^{n+m}(0,\vec{r}_1) \tilde{G}_0^n(\vec{r}_1,0) \right) U_{eff}(m,\vec{r}_2) L_{n,n+m}^{\alpha\alpha,\alpha\alpha}(\vec{r}-\vec{r}_1+\vec{r}_2) \end{aligned} \tag{A.8}$$

The above corrections are negligible in the parameter N^{-1} compared with the correction determined by equation (35).

The corrections to the chemical potential are such as

$$\begin{aligned} \frac{\delta\mu_1}{2\pi l^2} &= T \sum_m \int d^2\vec{r}\, P_N(0,\vec{r}) \tilde{G}_0^m(\vec{r},0) U_{eff}(m,\vec{r}) \\ \frac{\delta\mu_2}{2\pi l^2} &= -4T \sum_m \int d^2\vec{r}\, d^2\vec{r}_1\, d^2\vec{r}_2\, P_N(0,\vec{r}) \tilde{G}_0^m(\vec{r},0) U_{eff}(m,\vec{r}_1) \pi_0^0(m,\vec{r}_1-\vec{r}_2) L_{0,m}^{\alpha\alpha,\alpha\alpha}(\vec{r}_2-\vec{r}) \\ \frac{\delta\mu_3}{2\pi l^2} &= -8T^2 \sum_m \int d^2\vec{r}\, d^2\vec{r}_1\, d^2\vec{r}_2\, P_N(0,\vec{r}) \tilde{G}_0^m(\vec{r},0) U_{eff}(m,\vec{r}_1) L_{0,m}^{\alpha\alpha,\alpha\alpha}(\vec{r}-\vec{r}_1+\vec{r}_2) \\ &\times \left(G_0^0(0,\vec{r}_1) \tilde{G}_0^m(\vec{r}_1,0) + G_0^m(0,\vec{r}_1) \tilde{G}_0^0(\vec{r}_1,0) \right) \end{aligned} \tag{A.9}$$

The second and third corrections are negligible in the parameter N^{-1} compared with the first term. Hence the shift of the chemical potential $\delta\mu$ is determined mainly by the first correction $\delta\mu_1$.

9 Appendix B

In this appendix the evaluation of the polarization operator $\Pi(\omega_n, q)$ is presented. The condition $\omega_c\tau \gg 1$ is assumed to be hold. Then

$$\Pi(\omega_n, q) = T\sum_m \pi^m(\omega_n, q) \tag{B.1}$$

The calculation of the polarization operator $\Pi(\omega_n, q)$ is analogous to that given in Ref. [7]. The wave vectors $q \ll R_c/l^2$ are considered.

Using equation (31), one immediately obtains

$$\Pi(\zeta_n, Q) = \frac{2m}{\pi}\sum_{j=1}^{\infty}\frac{\mathcal{J}_j^2(Q)}{j^2+\zeta_n}\left[j^2 - \frac{1}{\pi\tau\omega_c}\mathcal{L}_j(\zeta_n)\Big)\right] \tag{B.2}$$

where

$$\mathcal{L}_j(\zeta_n) = \frac{j^2(j^2+3\zeta_n^2)}{\zeta_n^2(j^2+\zeta_n^2)}\ln\frac{\sinh\pi\zeta_n}{\pi\zeta_n} + 4\frac{j\zeta_n}{j^2+\zeta_n^2}\arctan\frac{\zeta_n}{j} + \frac{j^2-\zeta_n^2}{j^2+\zeta_n^2}\ln(1+2\tau\omega_c\zeta_n) \tag{B.3}$$

Here two parameters $\zeta_n = \omega/\omega_c$ ($\omega = 2\pi Tn$) and $Q = qR_c$ are introduced. The transformation of a series (B.2) into the integral form yields the asymptotic form of the polarization operator in the different regimes. In the static limit $\zeta_n \ll 1$

$$\Pi(\zeta_n, Q) = \frac{m}{\pi}\left(\left(1 - \frac{\pi}{6\tau\omega_c}\right)(1-\mathcal{J}_0^2(Q)) + \frac{\ln(1+2\omega_c\tau\zeta_n)}{2\pi\omega_c\tau}\xi(Q) + \mathcal{O}(\zeta_n^2)\right) \tag{B.4}$$

where the function $\xi(x)$ is defined as

$$\xi(x) = \int_0^{\pi}\frac{dy}{\pi}\mathcal{J}_0(2x\sin\frac{y}{2})\left[(y-\pi)^2 - \frac{\pi^2}{3}\right] \tag{B.5}$$

and its asymptotic form is given by

$$\xi(x) = \begin{cases} x^2 & , \quad x \ll 1 \\ \dfrac{\pi}{3x}(2-\sin 2x) & , \quad x \gg 1 \end{cases} \tag{B.6}$$

In the hydrodynamic limit $qR_c \ll 1$ we obtain

$$\Pi(\zeta_n, Q) \quad = \quad \frac{m}{2\pi}\frac{Q^2}{1+\zeta_n^2}\Big[1 - \frac{1}{\pi\tau\omega_c}\mathcal{L}_1(\zeta_n)\Big] \tag{B.7}$$

10 Appendix C

In this appendix the evaluation of the corrections to the thermodynamic and chemical potentials are presented.

10.1 Thermodynamic potential correction

Using equation (35), one can separate the thermodynamic potential correction into the exchange and correlation ones as follows

$$\Delta\Omega = \Delta\Omega_{ex} + \Delta\Omega_c \tag{C.1}$$

$$\frac{\Delta\Omega_{ex}}{L_xL_y} = \frac{T}{2}\sum_n \int \frac{d^2\vec{q}}{(2\pi)^2} U_0(q)\Pi(n,q) \tag{C.2}$$

$$\frac{\Delta\Omega_{c}}{L_xL_y} = -\frac{T}{2}\sum_n \int \frac{d^2\vec{q}}{(2\pi)^2} \int\limits_0^1 d\alpha \frac{\alpha U_0^2(q)\Pi^2(n,q)}{1+\alpha U_0(q)\Pi(n,q)} \tag{C.3}$$

The exchange correction gives the leading contribution [7] and can be written in the following way

$$\frac{\Delta\Omega_{ex}}{L_xL_y} = -\frac{e^2}{2\pi l^3}\sum_{m\neq N}\int\limits_0^\infty dx\, e^{-x^2/2} L_N^1(\frac{x^2}{2}) L_m(\frac{x^2}{2}) \left(\Theta(N-m) + \frac{1}{\pi\omega_c\tau}\frac{1}{m-N}\right) \tag{C.4}$$

where L_n^m stands for the Laguerre polynomials. The above equation in the case $N \gg 1$ goes over into equation (47).

10.2 Chemical potential correction

Using equation (37), one can separate the chemical potential correction onto exchange and correlation ones, respectively,

$$\delta\mu = \delta\mu_{ex} + \delta\mu_c \tag{C.5}$$

$$\delta\mu_{ex} = 2\pi l^2 T \sum_n \int d^2\vec{r} U_0(r) P_N(0,\vec{r}) \tilde{G}_0^n(\vec{r},0) \tag{C.6}$$

$$\delta\mu_c = -2\pi l^2 T \sum_n \int \frac{d^2\vec{q}}{(2\pi)^2} P_N(q) \tilde{G}_0^n(q) \frac{U_0^2(q)\Pi(n,q)}{1+U_0(q)\Pi(n,q)} \tag{C.7}$$

The exchange correction gives the leading contribution [7] and can be written in the following way

$$\delta\mu_{ex} = -\frac{e^2}{l}\sum_{m\neq N}\int\limits_0^\infty dx\, e^{-x^2/2} L_N(\frac{x^2}{2}) L_m(\frac{x^2}{2}) \left(\Theta(N-m) + \frac{1}{2\pi\omega_c\tau}\frac{1}{m-N}\right) \tag{C.8}$$

The above equation in the case $N \gg 1$ leads to

$$\delta\mu_{ex} = \frac{2e^2}{\pi l^2}(2N)^{1/2}\left[1 - \frac{\ln N}{8N} + \frac{1}{4\pi\omega_c\tau}\frac{1}{2N}\Big(\int\limits_1^\infty \frac{dt}{t}\ln(1-e^{-t}) + \int\limits_0^1 \frac{dt}{t}\ln\frac{1-e^{-t}}{t} - \frac{\pi^2}{3}\Big)\right] \tag{C.9}$$

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